Chapter 5 Practice Exercises (*Optional)

5-01 Fundamental Trigonometric Identities Part A

Use the given values to evaluate all six trigonometric functions.

1. $\sin\phi = \frac{2}{3}, \cos\phi = \frac{\sqrt{5}}{3}$
2. $\tan\theta = -\frac{7}{24}, \sec\theta = -\frac{25}{24}$
3. $\csc\alpha = -\frac{6}{5}, \cot\alpha = \frac{\sqrt{11}}{5}$
4. $\sin x = \frac{5}{13}, \tan x < 0$
5. $\tan y = undefined, \sin y < 0$

Match the trigonometric expression with one of the following.

(a) $2\sin^2 x - 1$	**(b) sin** x
(c) sec x	**(d) cos** x
(e) csc x	**(f) tan** x

6. $\cos x \tan x$
7. $(\sec x)\left(1 - \sin^2 x\right)$
8. $\sin^4 x - \cos^4 x$
9. $-\tan(-x)\sec\left(\frac{\pi}{2} - x\right)$
10. $\sin x + \sin x \cot^2 x$

Use the fundamental identities to simplify the expression. There may be more than one correct answer.

11. $\tan x \sin\left(\frac{\pi}{2} - x\right) + \cot x \sin^2 x \sec x$
12. $\tan\alpha\,(\cot\alpha + \tan\alpha)$
13. $\frac{\sin^2\theta}{1-\cos\theta}$
14. $\frac{1}{1+\cot^2\phi}$
15. $\frac{1-\sin^2 x}{\sec x}$

Mixed Review

16. (4-11) A ship leaves port and travels for 2 hours at 1.5 knots due south. Then it changes course due west for 1 hour. Find the distance and bearing from the starting point.
17. (4-10) A park is in the shape of a right triangle with the perpendicular side lengths 400 ft and 500 ft. What is the size of the acute angle adjacent to the 400 ft side (round to the nearest tenth)?
18. (3-02) Rewrite the logarithm in exponential form: $\log_3 81 = 4$.
19. (2-01) Divide $\frac{2+i}{1-i}$.
20. (1-08) If $f(x) = x^2 + 1$ and $g(x) = x - 4$, find $(f \circ g)(x)$.

5-02 Fundamental Trigonometric Identities Part B

Factor the expression, then use fundamental trigonometric identities to simplify.

1. $\sin^2 x \csc^2 x - \sin^2 x$
2. $\cot^4 x + 2\cot^2 x + 1$
3. $\tan^4 x - \sec^4 x$

Multiply and use trigonometric identities to simplify.

4. $(\cos x + \sin x)^2$
5. $(2\sec x + 2)(2\sec x - 2)$
6. $(\csc x - \cot x)(\csc x + \cot x)$

Add or subtract the expressions and use trigonometric identities to simplify.

7. $\frac{1}{1+\sin x} + \frac{1}{1-\sin x}$
8. $\frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x}$
9. $\cot x - \frac{\csc^2 x}{\cot x}$

Rewrite the fraction so it is not in fractional form.

10. $\frac{\cos^2 x}{1-\sin x}$
11. $\frac{4}{\sec x+\tan x}$

Use trigonometric substitution to write the algebraic expression as a trigonometric expression.

12. $\sqrt{4-x^2}; x = 2\sin\theta$
13. $\sqrt{x^2-36}; x = 6\csc\theta$

Rewrite the expression as a single logarithm.

14. $\ln|\sin x| + \ln|\csc x|$

Problem Solving

15. If an object slides down an inclined surface at a constant speed, the force of friction has to equal the component of the object's weight pulling it down the surface. The equation becomes

$$\mu W\cos\theta = W\sin\theta$$

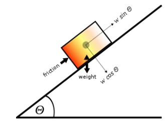


Figure 1: Object on inclined surface. (wikimedia/J.Spudeman)

where μ is the coefficient of friction, W is the weight, and θ is the incline of the surface. Solve the equation for μ and use fundamental trigonometric identities to simplify the expression.

Mixed Review

16. (5-01) Simplify $(1 - \cos^2 x)\csc x$.
17. (5-01) Given $\sin\theta = -\frac{\sqrt{5}}{5}$ and $\tan\theta < 0$, evaluate $\cos\theta$ and $\cot\theta$.
18. (4-05) Evaluate $\tan\frac{7\pi}{4}$ using reference angles.
19. (3-05) A substance has a half-life of 5.00 minutes. If the initial amount of the substance was 20.0 grams, how many half-lives will have passed before the substance decays to 13.0 grams?
20. (2-06) Find all the solutions of $0 = x^3 - 7x^2 + 17x - 15$.

5-03 Verify Trigonometric Identities

1. Derive the other two Pythagorean identities from $\sin^2 u + \cos^2 u = 1$.

Verify the identities.

2. $(1 - \cos t)(1 + \cos t) = \sin^2 t$
3. $(\csc\alpha + 1)(\csc\alpha - 1) = \cot^2\alpha$
4. $\cos x - \cos^3 x = \cos x \sin^2 x$
5. $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$
6. $\frac{\sin x-1}{\cos x} = \tan x - \sec x$
7. $\frac{\csc^2 x}{\tan x} = \cot x + \cot^3 x$
8. $\frac{1}{\cos x\sin x} = \cot x + \tan x$
9. $\tan(-t)\cos(-t)\cot(-t) = \cos t$
10. $\sin(-x)\csc\left(\frac{\pi}{2} - x\right) = -\tan x$
11. $\frac{\cos x+\cot y}{\cos x\cot y} = \sec x + \tan y$
12. $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = 2\sec^2 x$

Verify the identity algebraically and graphically.

13. $\tan\left(\frac{\pi}{2} - x\right)\tan x = 1$
14. $\sin^4 x = \sin^2 x - \sin^2 x\cos^2 x$
15. The length, ℓ, of a shadow cast by a vertical stick of height, h, when the angle of elevation of the sun is θ can be modeled by

$$\ell = \frac{h\cos\theta}{\cos(90^\circ - \theta)}$$

Verify that $\ell = h\cot\theta$.

Mixed Review

16. (5-02) Factor and then use the fundamental trigonometric identities to simplify: $\sin^4 x - 2\sin^2 x + 1$.
17. (5-02) Rewrite the fraction so it is not in fractional form: $\frac{3}{\csc x\tan x}$.
18. (5-01) Simplify: $\sin x(\csc x - \sin x)$.
19. (4-02) List all the angles, θ, on the unit circle where $\sin\theta = \pm\frac{\sqrt{3}}{2}$.
20. (3-03) Condense: $\log 2x + 3\log x - 4\log y$.

5-04 Solve Trigonometric Equations

1. Describe how to find all solutions in the interval $[0, 2\pi)$ when the angle is 3θ.

Find all solutions of the following equations.

2. $2\cos x - \sqrt{2} = 0$
3. $\csc x - 2 = 0$
4. $9\tan^2 x - 3 = 0$
5. $\sin x\cos x + \sin x = 0$
6. $\cos x - \cos^2 x = 0$
7. $\csc^2 x - 3\csc x + 2 = 0$

Find all solutions in the interval [0, 2π).

8. $2\sin^2 3x - \sin 3x - 1 = 0$
9. $\csc^2 x + \cot^2 x = 3$
10. $2\sin 2x\cot 2x = \sqrt{3}$
11. $\cot^2\left(\frac{\pi}{2} - 2x\right) = 3$
12. $\sin x + \cos x = 1$

Use a graphing utility to find the solutions in the interval [0, 2π) to three decimal places.

13. $\sin x + \cos^2 x = 0$
14. $\sec^2 x + \sec x - 2 = 0$

Problem Solving

15. When the initial height and final height of a projectile are equal, then the distance, or range, of the projectile is modeled by $R = \frac{v^2\sin 2\theta}{32}$ where R is the range in feet, v is the initial speed in feet per second, and θ is the launch angle. If a football receiver is 20 yards away from

the quarterback and the ball is thrown at 50 ft/s, at what angle should the ball be thrown?

Mixed Review

16. (5-03) Verify $\cos(-x)\sec\left(\frac{\pi}{2}-x\right)=\cot x$.
17. (5-03) Verify $\sin^3 x - \sin x = -\cos^2 x \sin x$.
18. (5-02) Simplify using identities: $(3 - 3\sin x)(3 + 3\sin x)$.
19. (5-01) Use the given values to evaluate all six trigonometric functions: $\sin x = \frac{7}{25}, \tan x < 0$.
20. (4-09) Find the exact value, if possible, without a calculator: $\sin\left(\cos^{-1}\frac{5}{13}\right)$.

5-05 Sum and Difference Formulas

Use a sum or difference formula to find the exact value of the expression.

1. $\sin 255°$
2. $\tan\frac{5\pi}{12}$
3. $\cos 230° \cos 20° + \sin 230° \sin 20°$
4. $\cos\frac{11\pi}{12}$

Derive a formula for the following expressions.

5. $\tan(x - \pi)$
6. $\sin\left(\frac{3\pi}{2}-x\right)$

Verify the following identities.

7. $\sec x \sin(\pi + x) = -\tan x$
8. $\tan(\pi + x)\sin\left(\frac{3\pi}{2}-x\right) = -\sin x$
9. $\frac{\cos(x+h)-\cos x}{h} = \cos x\left(\frac{\cos h - 1}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)$

Solve the trigonometric equations on the interval [0, 2π).

10. $\sin(2x - 3\pi) = 0$
11. $\cos\left(x+\frac{3\pi}{4}\right) - \cos\left(x-\frac{3\pi}{4}\right) = 1$
12. $\sin(\pi + x) - \sin(\pi - x) = 2$

Mixed Review

13. (5-04) Find all the solutions of $2\tan x + 1 = 3$.
14. (5-04) Use a graphing utility to find the solutions in the interval $[0, 2\pi)$ to three decimal places: $\sin x + \cos x = 1$.
15. (5-03) Verify the identity graphically: $\frac{\cos x - \cos^3 x}{\sin x} = \cos x \sin x$.

5-06 Multiple Angle Formulas

1. Derive a power-reducing formula from a double-angle formula.
2. If $\tan\theta = \frac{2}{3}$, find (a) sin 2θ, (b) cos 2θ, (c) tan 2θ.
3. If $\sin\alpha = \frac{7}{25}$, find (a) sin 2α, (b) cos 2α, (c) tan 2α.
4. *If $\cos\beta = \frac{1}{2}$, find (a) sin 2β, (b) cos 2β, (c) tan 2β.

Find all solutions on the interval [0, 2π).

5. $\sin^2 x + \cos 2x = 0$
6. $2\sin x\cos x = \frac{\sqrt{2}}{2}$
7. $\sin 2x - \cos x = 0$

Derive a new identity for the given expression.

8. $\cos 3x$
9. $\sin 4x$
10. *$\cos 4x$

Rewrite the expression as a sum of 1st powers of cosine.

11. $\cos^3 x$
12. $\cos^4 x$
13. $\tan^4 x$

Find the exact value of the following expressions.

14. *cos 15°
15. tan 75°
16. sin 105°

Find all the solutions.

17. $\cos\frac{x}{2} = \cos x$
18. $\tan\frac{x}{2} = \sin x$
19. *$\cos^2\frac{x}{2} = \cos x$

Mixed Review

20. (5-05) Derive a formula for $\cos(x + \pi)$.
21. (5-05) Solve $\cos\left(x-\frac{3\pi}{2}\right)+\cos\left(x+\frac{3\pi}{2}\right) = 0$ on $[0, 2\pi)$.
22. (5-04) Find all the solutions of $2\tan x + 3 = 1$.
23. (5-03) Verify $\frac{1-\sin^2 x}{\cos^2 x} = 1$.
24. (5-01) Simplify $\frac{\tan^2 x}{\sec x + 1}$.

5-07 Product-to-Sum Formulas

Rewrite the expression as a sum or difference.

1. sin 7α cos 2α
2. cos 7α sin 2α
3. $\sin 2x \sin x$
4. $\cos 3x \cos 2x$

Find the exact value of the expression.

5. sin 105° + sin 15°
6. cos 165° − cos 105°
7. sin 105° cos 15°
8. cos 285° cos 75°

Solve the equation on the interval [0, 2π).

9. $\cos 3x - \cos 2x = 0$
10. $\sin 4x = \sin x$
11. $\sin 2x \cos x = 0$
12. $\cos 2x + \cos x = 0$

Verify the identity.

13. $\frac{\sin 2x + \sin x}{\cos 2x + \cos x} = \tan\frac{3x}{2}$
14. $\frac{\sin 3x \cos 2x}{\cos 3x \sin 2x} = \frac{\sin 5x + \sin x}{\sin 5x - \sin x}$
15. $\frac{\cos 4x \cos x}{\sin 4x \sin x} = \frac{\cos 3x + \cos 5x}{\cos 3x - \cos 5x}$

Mixed Review

16. (5-06) Rewrite the expression as a sum of 1st powers of cosine: $\sin^2 x \cos^2 x$.
17. (5-06) If $\tan\theta = \frac{3}{4}$, find (a) $\sin\frac{\theta}{2}$, (b) $\cos\frac{\theta}{2}$, (c) $\tan\frac{\theta}{2}$.
18. (5-05) Verify $\tan(x + \pi)\cot x = 1$.
19. (5-03) Verify $\frac{1}{\sec x \csc x} = \tan x - \sin^2 x \tan x$.
20. (5-02) Simplify $\frac{\sin x}{1-\cos x} - \frac{\cos x}{\sin x}$.

5-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers. On this assignment round your answers to three decimal places unless otherwise directed.

1. If $\tan x = 1$ and $\cos x < 0$, find $\sin x$.
2. Simplify $(\tan^2 x + 1)(\cot^2 x + 1)$.
3. Simplify $\sec x - \sec x \sin^2 x$.
4. Solve on the interval $[0, 2\pi)$: $\sec^2 x = 1 + \tan x$.

Verify the identity.

5. $\cot x = \frac{\csc x \sec x}{1 + \tan^2 x}$
6. $\cos(x - \pi) = -\cos x$
7. $\sec\left(x-\frac{\pi}{2}\right)\cos(-x) = \cot x$
8. $\sin x \sin 2x = 2\cos x - 2\cos^3 x$
9. Use a power reducing formula to rewrite the following in terms of the first power of cosine: $\sin^4 x$.
10. If $\sin x = \frac{\sqrt{3}}{2}$ and $0 < x \le \frac{\pi}{2}$, find $\tan\frac{x}{2}$.
11. If $\sin\alpha = \frac{40}{41}$ and $\frac{\pi}{2} < \alpha < \pi$, find tan 2α.
12. Write $\cos 3x - \cos 2x$ as a product.
13. Write $\cos 3x \sin 2x$ as a sum or difference.

Solve on the interval [0, 2π).

14. $\cos 3x + \cos x = 0$
15. $\sin 2x \sec x = 2\sin 2x$
16. $2\cos x + \sqrt{3} = 0$
17. $3\tan 2x = \sqrt{3}$
18. $2\cos^2 x + 3\cos x + 1 = 0$
19. Use a graphing utility to approximate the solutions of $\tan x + \cos x = 0$ on the interval $[0, 2\pi)$. Round to 4 decimal places.
20. Find the exact value of tan 345° given that 345 = 135 + 210.
21. A baseball leaves the hand of the person at first base at an angle of α with the horizontal and at an initial velocity of $v_0 = 30$ meters per second. The ball is caught by another person 20 meters away. Find α if the range, r, of a projectile is $r = \frac{1}{32}v_0^2 \sin 2\alpha$. Use degrees.

Answers

5-01

1. $\tan\phi = \frac{2\sqrt{5}}{5}$, $\csc\phi = \frac{3}{2}$, $\sec\phi = \frac{3\sqrt{5}}{5}$, $\cot\phi = \frac{\sqrt{5}}{2}$
2. $\sin\theta = \frac{7}{25}$, $\cos\theta = -\frac{24}{25}$, $\csc\theta = \frac{25}{7}$, $\cot\theta = -\frac{24}{7}$
3. $\sin\alpha = -\frac{5}{6}$, $\cos\alpha = -\frac{\sqrt{11}}{6}$, $\tan\alpha = \frac{5\sqrt{11}}{11}$, $\sec\alpha = -\frac{6\sqrt{11}}{11}$
4. $\cos x = -\frac{12}{13}$, $\tan x = -\frac{5}{12}$, $\csc x = \frac{13}{5}$, $\sec x = -\frac{13}{12}$, $\cot x = -\frac{12}{5}$
5. $\sin y = -1$, $\cos y = 0$, $\csc y = -1$, $\sec y$ = undefined, $\cot y = 0$
6. b
7. d
8. a
9. c
10. e
11. 2 sin x
12. $\sec^2\alpha$
13. 1 + cos θ
14. $\sin^2\phi$
15. $\cos^3 x$
16. 3.35 nautical miles at S 26.6° W
17. 51.3°

18. $3^4 = 81$

19. $\frac{1+3i}{2}$

20. $x^2 - 8x + 17$

5-02

1. $\cos^2 x$
2. $\csc^4 x$
3. $-2 \tan^2 x - 1$
4. $2 \cos x \sin x + 1$
5. $4 \tan^2 x$
6. 1
7. $2 \sec^2 x$
8. $\sec x$
9. $-\tan x$
10. $1 + \sin x$
11. $4 \sec x - 4 \tan x$
12. $2 \cos \theta$
13. $6 \cot \theta$
14. 0
15. $\mu = \tan \theta$
16. $\sin x$
17. $\cos \theta = \frac{2\sqrt{5}}{5}$, $\cot \theta = -2$
18. -1
19. 0.621
20. $3, 2 + i, 2 - i$

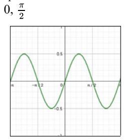
5-03

1. a) $\sin^2 u + \cos^2 u = 1 \rightarrow \frac{\sin^2 u+\cos^2 u}{\sin^2 u} = \frac{1}{\sin^2 u} \rightarrow 1 + \cot^2 u = \csc^2 u$; b) $\sin^2 u + \cos^2 u = 1 \rightarrow \frac{\sin^2 u+\cos^2 u}{\cos^2 u} = \frac{1}{\cos^2 u} \rightarrow \tan^2 u + 1 = \sec^2 u$
2. $(1 - \cos t)(1 + \cos t) = 1 - \cos^2 t = \sin^2 t$
3. $(\csc \alpha + 1)(\csc \alpha - 1) = \csc^2 \alpha - 1 = (\cot^2 \alpha + 1) - 1 = \cot^2 \alpha$
4. $\cos x - \cos^3 x = \cos x \left(1 - \cos^2 x\right) = \cos x \sin^2 x$
5. $\sin^4 x - \cos^4 x = \left(\sin^2 x - \cos^2 x\right)\left(\sin^2 x + \cos^2 x\right) = \left(\sin^2 x - \cos^2 x\right)(1) = \sin^2 x - \cos^2 x = 1 - \cos^2 x - \cos^2 x = 1 - 2\cos^2 x$
6. $\frac{\sin x-1}{\cos x} = \frac{\sin x}{\cos x} - \frac{1}{\cos x} = \tan x - \sec x$
7. $\frac{\csc^2 x}{\tan x} = \frac{1+\cot^2 x}{\tan x} = \frac{1}{\tan x} + \frac{\cot^2 x}{\tan x} = \cot x + \cot^3 x$
8. $\frac{1}{\cos x \sin x} = \frac{\cos x}{\cos^2 x \sin x} = \sec^2 x \cot x = \frac{\sec^2 x}{\tan x} = \frac{1+\tan^2 x}{\tan x} = \frac{1}{\tan x} + \frac{\tan^2 x}{\tan x} = \cot x + \tan x$
9. $\tan(-t)\cos(-t)\cot(-t) = \tan(-t)\cot(-t)\cos(-t) = \cos(-t) = \cos t$
10. $\sin(-x)\csc\left(\frac{\pi}{2} - x\right) = -\sin x \sec x = -\frac{\sin x}{\cos x} = -\tan x$
11. $\frac{\cos x+\cot y}{\cos x \cot y} = \frac{\cos x}{\cos x \cot y} + \frac{\cot y}{\cos x \cot y} = \frac{1}{\cot y} + \frac{1}{\cos x} = \sec x + \tan y$
12. $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = \frac{1+\sin x}{(1-\sin x)(1+\sin x)} + \frac{1-\sin x}{(1-\sin x)(1+\sin x)} = \frac{2}{1-\sin^2 x} = \frac{2}{\cos^2 x} = 2\sec^2 x$
13. $\tan\left(\frac{\pi}{2} - x\right)\tan x = \cot x \tan x = 1$
14. $\sin^4 x = \sin^2 x \sin^2 x = \sin^2 x \left(1 - \cos^2 x\right) = \sin^2 x - \sin^2 x \cos^2 x$
15. $\ell = \frac{h \cos \theta}{\cos(90^\circ - \theta)} = \frac{h \cos \theta}{\sin \theta} = h \cot \theta$
16. $\cos^4 x$
17. $3 \cos x$
18. $\cos^2 x$
19. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
20. $\log \frac{2x^4}{y^4}$

5-04

1. Find the expression for all the solutions and then plug in values for n starting with 0 until the angle is greater than 2π.
2. $\frac{\pi}{4} + 2\pi n, \frac{7\pi}{4} + 2\pi n$
3. $\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n$
4. $\frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n$
5. πn
6. $\frac{\pi}{2} + \pi n, 2\pi n$
7. $\frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \frac{\pi}{2} + 2\pi n$
8. $\frac{\pi}{6}, \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{5\pi}{6}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{3\pi}{2}, \frac{31\pi}{18}, \frac{35\pi}{18}$
9. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
10. $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$
11. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}$
12. $0, \frac{\pi}{2}$
13. 3.808, 5.617
14. 0, 2.094, 4.189
15. 25.1°, 64.9°
16. $\cos(-x)\sec\left(\frac{\pi}{2} - x\right) = \cos x \csc x = \frac{\cos x}{\sin x} = \cot x$
17. $\sin^3 x - \sin x = \sin x \left(\sin^2 x - 1\right) = \sin x \left(1 - \cos^2 x - 1\right) = \sin x \left(-\cos^2 x\right) = -\cos^2 x \sin x$
18. $9 \cos^2 x$
19. $\cos x = -\frac{24}{25}$, $\tan x = -\frac{7}{24}$, $\csc x = \frac{25}{7}$, $\sec x = -\frac{25}{24}$, $\cot x = -\frac{24}{7}$
20. $\frac{12}{13}$

5-05

1. $\frac{-\sqrt{6}-\sqrt{2}}{4}$
2. $\sqrt{3} + 2$
3. $-\frac{\sqrt{3}}{2}$
4. $\frac{-\sqrt{6}-\sqrt{2}}{4}$
5. $\tan x$
6. $-\cos x$
7. $\sec x \sin(\pi + x) = \sec x \left(\sin \pi \cos x + \cos \pi \sin x\right) = \sec x \left(-\sin x\right) = -\tan x$
8. $\tan(\pi + x)\sin\left(\frac{3\pi}{2} - x\right) = \left(\frac{\tan \pi+\tan x}{1-\tan \pi \tan x}\right)\left(\sin \frac{3\pi}{2}\cos x - \cos \frac{3\pi}{2}\sin x\right) = (\tan x)(-\cos x) = -\sin x$
9. $\frac{\cos(x+h)-\cos x}{h} = \frac{\cos x \cos h-\sin x \sin h-\cos x}{h} = \frac{\cos x(\cos h-1)-\sin x \sin h}{h} = \cos x\left(\frac{\cos h-1}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)$
10. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
11. $\frac{5\pi}{4}, \frac{7\pi}{4}$
12. $\frac{3\pi}{2}$
13. $\frac{\pi}{4} + \pi n$
14. $0, \frac{\pi}{2}$
15. 

5-06

1. $\cos 2u = 2\cos^2 u - 1 \rightarrow 1 + \cos 2u = 2\cos^2 u \rightarrow \frac{1+\cos 2u}{2} = \cos^2 u$
2. (a) $\frac{12}{13}$; (b) $\frac{5}{13}$; (c) $\frac{12}{5}$
3. (a) $\frac{336}{625}$; (b) $\frac{527}{625}$; (c) $\frac{336}{527}$
4. (a) $\frac{\sqrt{3}}{2}$; (b) $-\frac{1}{2}$; (c) $-\sqrt{3}$
5. $\frac{\pi}{2}, \frac{3\pi}{2}$
6. $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$
7. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
8. $\cos 3x = 4\cos^3 x - 3\cos x$
9. $\sin 4x = 8\cos^3 x \sin x - 4\cos x \sin x$
10. $\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$
11. $\frac{1}{2}(\cos x + \cos 2x \cos x)$
12. $\frac{1}{8}(3 + 4\cos 2x + \cos 4x)$
13. $\frac{3-4\cos 2x+\cos 4x}{3+4\cos 2x+\cos 4x}$
14. $\frac{\sqrt{2+\sqrt{3}}}{2}$
15. $2 + \sqrt{3}$
16. $\frac{\sqrt{2+\sqrt{3}}}{2}$
17. $2\pi n, \frac{4\pi}{3} + 2\pi n$
18. $\frac{\pi}{2}n$
19. $2\pi n$
20. $-\cos x$
21. All real numbers
22. $\frac{3\pi}{4} + \pi n$
23. $\frac{1-\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1$
24. $\sec x - 1$

5-07

1. $\frac{1}{2}(\sin 9\alpha + \sin 5\alpha)$
2. $\frac{1}{2}(\sin 9\alpha - \sin 5\alpha)$
3. $\frac{1}{2}(\cos x - \cos 3x)$
4. $\frac{1}{2}(\cos x + \cos 5x)$
5. $\frac{\sqrt{6}}{2}$
6. $-\frac{\sqrt{2}}{2}$
7. $\frac{\sqrt{3}+2}{4}$
8. $\frac{-\sqrt{3}+2}{4}$
9. $0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$
10. $0, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{7\pi}{5}, \frac{9\pi}{5}$
11. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
12. $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$
13. $\frac{\sin 2x-\sin x}{\cos 2x+\cos x} = \frac{2\sin\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right)}{2\cos\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right)} = \frac{\sin \frac{3x}{2}}{\cos 3x2} = \tan \frac{3x}{2}$
14. $\frac{\sin 3x \cos 2x}{\cos 3x \sin 2x} = \frac{\frac{1}{2}(\sin(3x+2x)+\sin(3x-2x))}{\frac{1}{2}(\sin(3x+2x)-\sin(3x-2x))} = \frac{\sin 5x+\sin x}{\sin 5x-\sin x}$
15. $\frac{\cos 4x \cos x}{\sin 4x \sin x} = \frac{\frac{1}{2}(\cos(4x-x)+\cos(4x+x))}{\frac{1}{2}(\cos(4x-x)-\cos(4x+x))} = \frac{\cos 3x+\cos 5x}{\cos 3x-\cos 5x}$
16. $\frac{1-\cos 4x}{8}$
17. (a) $\frac{\sqrt{10}}{10}$, (b) $\frac{3\sqrt{10}}{10}$, (c) $\frac{1}{3}$
18. $\tan(x + \pi)\cot x = \left(\frac{\tan x+\tan \pi}{1-\tan x \tan \pi}\right)\cot x = \left(\frac{\tan x+0}{1-(\tan x)(0)}\right)\cot x = \tan x \cot x = 1$
19. $\frac{1}{\sec x \csc x} = \frac{\sec x}{\sec^2 x \csc x} = \cos^2 x \frac{\sin x}{\cos x} = \left(1 - \sin^2 x\right)\tan x = \tan x - \sin^2 x \tan x$
20. $\csc x$

5-Review

1. $-\frac{\sqrt{2}}{2}$
2. $\sec^2 x \csc^2 x$
3. $\cos x$
4. $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$
5. $\cot x = \frac{\csc x}{\sec x} = \frac{\csc x \sec x}{\sec^2 x} = \frac{\csc x \sec x}{1+\tan^2 x}$
6. $\cos(x - \pi) = \cos x \cos \pi + \sin x \sin \pi = -\cos x$
7. $\sec\left(x - \frac{\pi}{2}\right)\cos(-x) = \csc x (\cos x) = \frac{\cos x}{\sin x} = \cot x$
8. $\sin x \sin 2x = 2\sin x \cos x \sin x = 2\sin^2 x \cos x = 2\left(1 - \cos^2 x\right)\cos x = 2\cos x - 2\cos^3 x$
9. $\frac{3-4\cos 2x+\cos 4x}{8}$
10. $\frac{\sqrt{3}}{3}$
11. $\frac{720}{1519}$
12. $-2\sin \frac{5x}{2}\sin \frac{x}{2}$
13. $\frac{1}{2}(\sin 5x - \sin x)$
14. $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$
15. $0, \frac{\pi}{3}, \frac{5\pi}{3}, \pi$
16. $\frac{5\pi}{6}, \frac{7\pi}{6}$
17. $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$
18. $\frac{2\pi}{3}, \pi, \frac{4\pi}{3}$
19. 3.8078, 5.6169
20. $-2 + \sqrt{3}$
21. 22.7°, 67.3°